

Physical interpretation of generalized two-mode squeezing operator revealed by virtue of the transformation of entangled state representation*

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Abstract

By virtue of the integration method within \mathfrak{P} -ordered product of operators and the property of entangled state representation, we reveal new physical interpretation of the generalized two-mode squeezing operator (GTSO), and find it be decomposed as the product of free-space propagation operator, single-mode and two-mode squeezing operators, as well as thin lens transformation operator. This decomposition is useful to design of optical devices for generating various squeezed states of light. Transformation of entangled state representation induced by GTSO is emphasized.

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1 Introduction

In recent decades, nonclassical optical field (such as squeezed light field [1] and non-Gaussian optical field [2, 3, 4]) has been received much attention by physicists as it can be applied to quantum communication protocols [17, 18], metrology [5, 6, 7], cloning [8], quantum computation [19, 20], [14, 15] testing of quantum theory [21] and high precision quantum measurement, due to the fact that quadrature measurement of a squeezed state can be less than that of a coherent state, [5, 6, 7]. Based on the squeezed sideband mechanism, the squeezed light can also be applied in quantum teleportation [9]. Moreover, squeezed optical field has proven itself to be quite adapt to quantum key distribution [13], quantum swapping [11], controlled quantum dense encoding [12], and quantum phase tracking [16]. Theoretical analysis yields the general conclusion that entangled resource applied in continuous variable quantum communication is multi-mode squeezed light field. Experimentally, signal mode and idle mode of the output of a non-degenerate parametric amplifier constitutes a two-mode squeezed state.

Theoretically, the two-mode squeezing operator S_2 can be viewed as the quantum image of mapping of classical scale transformation $\eta \rightarrow \eta/\mu$ in the bipartite entangled state representation, and has a concise expression [26]

$$S_2 = \int \frac{d^2\eta}{\pi\mu} |\eta/\mu\rangle \langle \eta| = e^{\lambda(a_1^\dagger a_2^\dagger - a_1 a_2)}, \quad (1)$$

where $\mu = e^\lambda$ is squeezing parameter, $|\eta\rangle$ [25] is defined as

$$|\eta\rangle = \exp \left[-\frac{|\eta|^2}{2} + \eta a_1^\dagger - \eta^* a_2^\dagger + a_1^\dagger a_2^\dagger \right] |00\rangle, \quad \eta = \eta_1 + i\eta_2,$$

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which is also named Einstein-Podolsky-Rosen (EPR) entangled states, satisfying eigenvector equations

$$(Q_1 - Q_2) |\eta\rangle = \sqrt{2}\eta_1 |\eta\rangle, \quad (P_1 + P_2) |\eta\rangle = \sqrt{2}\eta_2 |\eta\rangle$$

where $Q_i = (a_i + a_i^\dagger)/\sqrt{2}$ and $P_i = i(a_i^\dagger - a_i)/\sqrt{2}$ are a pair of conjugate variables with $[Q_i, P_j] = i\delta_{ij}$, $\hbar = 1$. S_2 makes $\frac{Q_1+Q_2}{\sqrt{2}}$ and $\frac{P_1+P_2}{\sqrt{2}}$ as scaling transformation

$$S_2 \frac{Q_1 + Q_2}{\sqrt{2}} S_2^{-1} = \mu \frac{Q_1 + Q_2}{\sqrt{2}}, \quad S_2 \frac{P_1 + P_2}{\sqrt{2}} S_2^{-1} = \frac{P_1 + P_2}{\sqrt{2}\mu}. \quad (2)$$

An interesting question thus naturally arises: Is there a generalized two-mode squeezing operator, denoted by F_2 , which is responsible for generating the following transformations

$$\begin{aligned} F_2 (Q_1 + Q_2) F_2^{-1} &= A (Q_1 + Q_2) + C (P_1 + P_2) \\ F_2 (P_1 + P_2) F_2^{-1} &= D (P_1 + P_2) + B (Q_1 + Q_2) \end{aligned} \quad (3)$$

in which $AD - BC = 1$, and correspondingly,

$$\begin{aligned} F_2 (P_1 - P_2) F_2^{-1} &= A (P_1 - P_2) - C (Q_1 - Q_2) \\ F_2 (Q_1 - Q_2) F_2^{-1} &= D (Q_1 - Q_2) - B (P_1 - P_2), \end{aligned} \quad (4)$$

and what is the corresponding squeezed state? In this work, in order to derive the concrete expression of F_2 , in Sec. II we shall discuss transformation of the entangled state representation induced by F_2 . In Sec. III, by virtue of the integration method within \mathfrak{P} -ordered product of operators (which means that all momentum operators are arranged on the left of all coordinate operators) we shall show that F_2 can be decomposed as a product of free-space propagation operator, single-mode and two-mode squeezing operator, and thin lens transformation operator. In so doing, a new physical interpretation of the generalized two-mode squeezing operator is presented. This decomposition is useful to design of optical devices for generating various squeezed states of light. In Sec. IV, we shall examine the Lie algebra structure of F_2 . In the whole text, we shall make full use of properties of the entangled state representation.

2 Representation transformation caused by F_2

By introducing another bipartite entangled states $|\xi\rangle$ in Fock space [26]

$$|\xi\rangle = \exp \left[-\frac{|\xi|^2}{2} + \xi a_1^\dagger + \xi^* a_2^\dagger - a_1^\dagger a_2^\dagger |00\rangle \right], \quad \xi = \xi_1 + i\xi_2, \quad (5)$$

which is the eigenvector of $(P_1 - P_2)$ and $(Q_1 + Q_2)$

$$(Q_1 + Q_2) |\xi\rangle = \sqrt{2}\xi_1 |\xi\rangle, \quad (P_1 - P_2) |\xi\rangle = \sqrt{2}\xi_2 |\xi\rangle, \quad (6)$$

we see the inner product

$$\langle \xi | \eta \rangle = \frac{1}{2} e^{i(\eta_2 \xi_1 - \eta_1 \xi_2)} = \frac{1}{2} e^{(\eta \xi^* - \xi \eta^*)/2}, \quad (7)$$

so $|\xi\rangle$ is the conjugate state to $|\eta\rangle$. Since

$$[D(Q_1 - Q_2) - B(P_1 - P_2), D(P_1 + P_2) + B(Q_1 + Q_2)] = 0$$

they should possess common eigenvector, denoted as $|\eta\rangle_{D,B}$, obeying

$$[D(Q_1 - Q_2) - B(P_1 - P_2)] |\eta\rangle_{D,B} = \sqrt{2}\eta_1 |\eta\rangle_{D,B}, \quad (8)$$

$$[B(Q_1 + Q_2) + D(P_1 + P_2)] |\eta\rangle_{D,B} = \sqrt{2}\eta_2 |\eta\rangle_{D,B}. \quad (9)$$

From Eq. (5), we know

$$\langle \xi | (Q_1 - Q_2) = i\sqrt{2} \frac{\partial}{\partial \xi_2} \langle \xi |, \quad \langle \xi | (P_1 + P_2) = -i\sqrt{2} \frac{\partial}{\partial \xi_1} \langle \xi |. \quad (10)$$

Thus from (8) and (10), we have

$$\langle \xi | [D(Q_1 - Q_2) - B(P_1 + P_2)] | \eta \rangle_{D,B} = \sqrt{2} \eta_1 \langle \xi | \eta \rangle_{D,B} = \sqrt{2} \left(Di \frac{\partial}{\partial \xi_2} - B \xi_2 \right) \langle \xi | \eta \rangle_{D,B}, \quad (11)$$

its solution is

$$\langle \xi | \eta \rangle_{D,B} = \mathfrak{C}_1(\xi_1, \eta) \exp \left[-\frac{i}{D} \left(\xi_2 \eta_1 + \frac{B \xi_2^2}{2} \right) \right], \quad (12)$$

where $\mathfrak{C}_1(\xi_1, \eta)$ is an integral constant. In the same way, from (9) we can obtain

$$\langle \xi | [B(Q_1 + Q_2) + D(P_1 + P_2)] | \eta \rangle_{D,B} = \sqrt{2} \eta_2 \langle \xi | \eta \rangle_{D,B} = \sqrt{2} \left(B \xi_1 - Di \frac{\partial}{\partial \xi_1} \right) \langle \xi | \eta \rangle_{D,B}$$

with the solution

$$\langle \xi | \eta \rangle_{D,B} = \mathfrak{C}_2(\xi_2, \eta) \exp \left[\frac{i}{D} \left(\xi_1 \eta_2 - \frac{B \xi_1^2}{2} \right) \right]. \quad (13)$$

Eqs. (12) and (13) tell us

$$\langle \xi | \eta \rangle_{D,B} = \mathfrak{C}(\eta) \exp \left[\frac{i}{D} \left(\xi_1 \eta_2 - \xi_2 \eta_1 - \frac{B |\xi|^2}{2} \right) \right], \quad (14)$$

$\mathfrak{C}(\eta)$ is an integral constant. Using the completeness relation of $|\xi\rangle$, i.e. $\int \frac{d^2 \xi}{\pi} |\xi\rangle \langle \xi| = 1$, we have

$$\begin{aligned} |\eta\rangle_{D,B} &= \int \frac{d^2 \xi}{\pi} |\xi\rangle \langle \xi | \eta \rangle_{D,B} \\ &= \mathfrak{C}(\eta) \frac{2D}{D+iB} \exp \left\{ \frac{1}{D+iB} \left[\frac{-|\eta|^2}{2D} + \eta a_1^\dagger - \eta^* a_2^\dagger + (D-iB) a_1^\dagger a_2^\dagger \right] \right\} |00\rangle. \end{aligned}$$

If setting $\mathfrak{C}(\eta) = \frac{1}{2D} \exp \left[\frac{iC}{2D} |\eta|^2 \right]$, we obtain

$$|\eta\rangle_{D,B} = \frac{1}{D+iB} \exp \left[-\frac{A-iC}{2(D+iB)} |\eta|^2 + \frac{\eta a_1^\dagger}{D+iB} - \frac{\eta^* a_2^\dagger}{D+iB} + \frac{D-iB}{D+iB} a_1^\dagger a_2^\dagger \right],$$

which is complete, $\int \frac{d^2 \eta}{\pi} |\eta\rangle_{D,B} {}_{D,B} \langle \eta| = 1$. Consulting (3) and (4) and using orthogonality of the entangled state representation $|\eta\rangle$, $\langle \eta | \eta' \rangle = \pi \delta^{(2)}(\eta - \eta')$, we know that F_2 can be expressed as

$$F_2 = \int \frac{d^2 \eta}{\pi} |\eta\rangle_{D,B} {}_{D,B} \langle \eta|. \quad (15)$$

Thus the unitary operator F_2 can convert $|\eta\rangle$ into $|\eta\rangle_{D,B}$, (15) showing the representation transformation caused by F_2 .

3 Explicit Expression of F_2 and its decomposition

Considering the completeness relation $\int \frac{d^2 \xi}{\pi} |\xi\rangle \langle \xi| = 1$ and noting (7) we have

$$\begin{aligned} \frac{1}{2} |\xi\rangle \langle \eta| e^{-(\xi \eta^* - \eta \xi^*)/2} &= \pi^2 \delta \left(\xi_2 - \frac{P_1 - P_2}{\sqrt{2}} \right) \delta \left(\xi_1 - \frac{Q_1 + Q_2}{\sqrt{2}} \right) \\ &\quad \times \delta \left(\eta_2 - \frac{P_1 + P_2}{\sqrt{2}} \right) \delta \left(\eta_1 - \frac{Q_1 - Q_2}{\sqrt{2}} \right). \end{aligned}$$

Then using Eq. (14) and (15) we derive

$$\begin{aligned} F_2 &= \int \frac{d^2 \eta}{\pi} \int \frac{d^2 \xi}{\pi} |\xi\rangle \langle \xi | \eta \rangle_{D,B} {}_{D,B} \langle \eta| = \int \frac{d^2 \eta}{\pi} \int \frac{d^2 \xi}{\pi} \frac{1}{2} |\xi\rangle \langle \eta| e^{-(\xi \eta^* - \eta \xi^*)/2} \\ &\quad \times \frac{1}{D} \exp \left[i \frac{C |\eta|^2 - B |\xi|^2}{2D} + \frac{i(\xi_1 \eta_2 - \xi_2 \eta_1)(1-D)}{D} \right] \\ &= \frac{1}{D} \int d^2 \eta \int d^2 \xi \delta \left(\xi_2 - \frac{P_1 - P_2}{\sqrt{2}} \right) \delta \left(\xi_1 - \frac{Q_1 + Q_2}{\sqrt{2}} \right) \delta \left(\eta_2 - \frac{P_1 + P_2}{\sqrt{2}} \right) \\ &\quad \times \delta \left(\eta_1 - \frac{Q_1 - Q_2}{\sqrt{2}} \right) \exp \left[i \frac{C |\eta|^2 - B |\xi|^2}{2D} + \frac{i(\xi_1 \eta_2 - \xi_2 \eta_1)(1-D)}{D} \right]. \end{aligned} \quad (16)$$

Recalling that in Ref. [27, 28, 29, 30] the authors has proposed a new approach for handling \mathfrak{Q} -ordering (all Q are on the left of all P) and \mathfrak{P} -ordering (all P are on the left of all Q). By virtue of the integration method within \mathfrak{P} -ordered product of operator we have

$$\delta\left(\xi_1 - \frac{Q_1 + Q_2}{\sqrt{2}}\right) \delta\left(\eta_2 - \frac{P_1 + P_2}{\sqrt{2}}\right) = \frac{1}{2\pi} \mathfrak{P} \left[e^{i\left(\eta_2 - \frac{P_1 + P_2}{\sqrt{2}}\right)\left(\xi_1 - \frac{Q_1 + Q_2}{\sqrt{2}}\right)} \right]. \quad (17)$$

Thus the last equation of Eq. (16) reads as

$$\begin{aligned} F_2 &= \frac{1}{2\pi D} \int d^2\eta \int d^2\xi \delta\left(\xi_2 - \frac{P_1 - P_2}{\sqrt{2}}\right) \mathfrak{P} \left[e^{i\left(\eta_2 - \frac{P_1 + P_2}{\sqrt{2}}\right)\left(\xi_1 - \frac{Q_1 + Q_2}{\sqrt{2}}\right)} \right] \delta\left(\eta_1 - \frac{Q_1 - Q_2}{\sqrt{2}}\right) \\ &\quad \times \exp \left[i \frac{C|\eta|^2 - B|\xi|^2}{2D} + \frac{i(\xi_1\eta_2 - \xi_2\eta_1)(1-D)}{D} \right] \\ &= \frac{1}{2\pi D} \mathfrak{P} \left\{ \int d\eta_2 \int d\xi_1 e^{i\left(\eta_2 - \frac{P_1 + P_2}{\sqrt{2}}\right)\left(\xi_1 - \frac{Q_1 + Q_2}{\sqrt{2}}\right)} \exp \left[i \frac{C\eta_2^2 + C\frac{(Q_1 - Q_2)^2}{2} - B\xi_1^2 - B\frac{(P_1 - P_2)^2}{2}}{2D} \right. \right. \\ &\quad \left. \left. + \frac{i\left(\xi_1\eta_2 - \frac{P_1 - P_2}{\sqrt{2}}\frac{Q_1 - Q_2}{\sqrt{2}}\right)(1-D)}{D} \right] \right\}. \end{aligned}$$

Then performing integration over $d\xi_1$ and $d\eta_2$, we obtain

$$\begin{aligned} F_2 &= \sqrt{\frac{1}{AD}} \exp \left[\frac{iC}{4A} (P_1 + P_2)^2 - \frac{iB}{4D} (P_1 - P_2)^2 \right] \\ &\quad \times \mathfrak{P} \exp \left\{ -i(P_1 \ P_2) \begin{pmatrix} \frac{1}{2A} + \frac{1}{2D} - 1 & \frac{1}{2A} - \frac{1}{2D} \\ \frac{1}{2A} - \frac{1}{2D} & \frac{1}{2A} + \frac{1}{2D} - 1 \end{pmatrix} \begin{pmatrix} Q_1 \\ Q_2 \end{pmatrix} \right\} \\ &\quad \times \exp \left[\frac{iC}{4D} (Q_1 - Q_2)^2 - \frac{iB}{4A} (Q_1 + Q_2)^2 \right]. \end{aligned} \quad (18)$$

Then using the operator identity [29]

$$e^{-i\vec{P}\Lambda\vec{Q}} \equiv e^{(-iP)_l \Lambda_{lk} Q_k} = \mathfrak{P} [\exp\{(-iP)_l (e^\Lambda - 1)_{lk} Q_k\}] = \mathfrak{P} e^{-i\vec{P}(e^\Lambda - 1)\vec{Q}} \quad (19)$$

and

$$[(P_1 Q_1 + P_2 Q_2), (P_1 Q_2 + P_2 Q_1)] = 0,$$

we have

$$\begin{aligned} &\mathfrak{P} \exp\left\{-i(P_1 \ P_2) \begin{pmatrix} \frac{1}{2A} + \frac{1}{2D} - 1 & \frac{1}{2A} - \frac{1}{2D} \\ \frac{1}{2A} - \frac{1}{2D} & \frac{1}{2A} + \frac{1}{2D} - 1 \end{pmatrix} \begin{pmatrix} Q_1 \\ Q_2 \end{pmatrix}\right\} \\ &= \exp \left[-i(P_1 \ P_2) \begin{pmatrix} -\ln\sqrt{AD} & \ln\sqrt{\frac{D}{A}} \\ \ln\sqrt{\frac{D}{A}} & -\ln\sqrt{AD} \end{pmatrix} \begin{pmatrix} Q_1 \\ Q_2 \end{pmatrix} \right] \\ &= \exp \left[i(P_1 Q_1 + P_2 Q_2) \ln\sqrt{AD} \right] \exp \left[-i(P_1 Q_2 + P_2 Q_1) \ln\sqrt{\frac{D}{A}} \right]. \end{aligned} \quad (20)$$

Noting

$$\ln \begin{pmatrix} \frac{1}{2A} + \frac{1}{2D} & \frac{1}{2A} - \frac{1}{2D} \\ \frac{1}{2A} - \frac{1}{2D} & \frac{1}{2A} + \frac{1}{2D} \end{pmatrix} = \begin{pmatrix} -\ln\sqrt{AD} & \ln\sqrt{\frac{D}{A}} \\ \ln\sqrt{\frac{D}{A}} & -\ln\sqrt{AD} \end{pmatrix} \quad (21)$$

and in reference to Eq. (20), we consider the following transformation

$$\begin{aligned} e^{-i(P_1 Q_2 + P_2 Q_1) \ln\sqrt{\frac{D}{A}}} (Q_1 + Q_2) e^{i(P_1 Q_2 + P_2 Q_1) \ln\sqrt{\frac{D}{A}}} &= \sqrt{\frac{A}{D}} (Q_1 + Q_2), \\ e^{-i(P_1 Q_2 + P_2 Q_1) \ln\sqrt{\frac{D}{A}}} (Q_1 - Q_2) e^{i(P_1 Q_2 + P_2 Q_1) \ln\sqrt{\frac{D}{A}}} &= \sqrt{\frac{D}{A}} (Q_1 - Q_2), \\ e^{-i(P_1 Q_2 + P_2 Q_1) \ln\sqrt{\frac{D}{A}}} (P_1 + P_2) e^{i(P_1 Q_2 + P_2 Q_1) \ln\sqrt{\frac{D}{A}}} &= \sqrt{\frac{D}{A}} (P_1 + P_2), \end{aligned} \quad (22)$$

which indicates $e^{-i(P_1 Q_2 + P_2 Q_1) \ln \sqrt{\frac{D}{A}}}$ is a two-mode squeezing operator and

$$\begin{aligned} e^{i(P_1 Q_1 + P_2 Q_2) \ln \sqrt{AD}} (Q_1 \pm Q_2) e^{-i(P_1 Q_1 + P_2 Q_2) \ln \sqrt{AD}} &= \sqrt{AD} (Q_1 \pm Q_2), \\ e^{i(P_1 Q_1 + P_2 Q_2) \ln \sqrt{AD}} (P_1 \pm P_2) e^{-i(P_1 Q_1 + P_2 Q_2) \ln \sqrt{AD}} &= \frac{1}{\sqrt{AD}} (P_1 \pm P_2). \end{aligned} \quad (23)$$

Besides, in reference to Eq. (18), we see

$$\exp \left[-\frac{iB}{4A} (Q_1 + Q_2)^2 \right] (P_1 + P_2) \exp \left[\frac{iB}{4A} (Q_1 + Q_2)^2 \right] = P_1 + P_2 + \frac{B}{A} (Q_1 + Q_2).$$

Combining the last three results leads to Eqs. (3) and (4). Therefore, from Eqs. (18) and (20) we conclude that

$$\begin{aligned} F_2 &= \sqrt{\frac{1}{AD}} \exp \left[\frac{iC}{4A} (P_1 + P_2)^2 - \frac{iB}{4D} (P_1 - P_2)^2 \right] \\ &\times \exp \left[i(P_1 Q_1 + P_2 Q_2) \ln \sqrt{AD} \right] \exp \left[-i(P_1 Q_2 + P_2 Q_1) \ln \sqrt{\frac{D}{A}} \right] \\ &\times \exp \left[\frac{iC}{4D} (Q_1 - Q_2)^2 - \frac{iB}{4A} (Q_1 + Q_2)^2 \right]. \end{aligned} \quad (24)$$

is we seeked for. In Eq. (24), according to the terminology in matrix optics theory, the first term denotes a freespace propagation operator, the second and the third term are respectively the single-mode and two-mode squeezing operator, and last term is a thin lens transformation operator. This decomposition is useful to design of optical devices for generating various squeezed states of light.

4 Lie algebra of Equation (24)

In order to further explain that Eq. (24) is really a new decomposition, we study its ingredient

$$\begin{aligned} &\sqrt{\frac{1}{AD}} \exp \left[-\frac{iB}{4D} (P_1 - P_2)^2 \right] \exp \left[i(P_1 Q_1 + P_2 Q_2) \ln \sqrt{AD} \right] \\ &\times \exp \left[-i(P_1 Q_2 + P_2 Q_1) \ln \sqrt{\frac{D}{A}} \right] \exp \left[\frac{iC}{4D} (Q_1 - Q_2)^2 \right] \equiv G. \end{aligned} \quad (25)$$

We can prove

$$G = \exp \left\{ \frac{iC}{4A} (Q_1 - Q_2)^2 \right\} \exp [i(Q_1 P_2 + Q_2 P_1) \ln A] \exp \left\{ \frac{-iB}{4A} (P_1 - P_2)^2 \right\} \quad (26)$$

so we can convert Eq. (24) into

$$\begin{aligned} F_2 &= \exp \left\{ \frac{iC}{4A} [(Q_1 - Q_2)^2 + (P_1 + P_2)^2] \right\} \exp [i(Q_1 P_2 + Q_2 P_1) \ln A] \\ &\times \exp \left\{ \frac{-iB}{4A} [(Q_1 + Q_2)^2 + (P_1 - P_2)^2] \right\}. \end{aligned} \quad (27)$$

its merit lies in the obvious SU(1,1) Lie algebra structure

$$\begin{aligned} \left[\frac{(Q_1 - Q_2)^2 + (P_1 + P_2)^2}{4}, \frac{(Q_1 + Q_2)^2 + (P_1 - P_2)^2}{4} \right] &= \frac{-i}{2} (Q_1 P_2 + Q_2 P_1), \\ \left[\frac{-i}{2} (Q_1 P_2 + Q_2 P_1), \frac{(Q_1 - Q_2)^2 + (P_1 + P_2)^2}{4} \right] &= \frac{(Q_1 - Q_2)^2 + (P_1 + P_2)^2}{4}, \\ \left[\frac{-i}{2} (Q_1 P_2 + Q_2 P_1), \frac{(Q_1 + Q_2)^2 + (P_1 - P_2)^2}{4} \right] &= -\frac{(Q_1 + Q_2)^2 + (P_1 - P_2)^2}{4}, \end{aligned} \quad (28)$$

thus F_2 embodies $SU(1,1)$ Lie algebra. Now we show that the equivalence between Eq. (26) and Eq. (25) can be derived by using the integration method within \mathfrak{P} -ordered product of operators. In fact, using the entangled state representation $|\xi\rangle$, we have

$$\exp[i(Q_1 P_2 + Q_2 P_1) \ln A] |\xi\rangle = \frac{1}{A} |\xi/A\rangle \quad (29)$$

and using the completeness relation of $|\xi\rangle$ and $|\eta\rangle$, we can see

$$\begin{aligned} (26) &= \exp\left[\frac{iC}{4A}(Q_1 - Q_2)^2\right] \int \frac{d^2\eta}{\pi} |\eta\rangle \langle\eta| \exp[i(Q_1 P_2 + Q_2 P_1) \ln A] \int \frac{d^2\xi}{\pi} |\xi\rangle \langle\xi| \exp\left[\frac{-iB}{4A}(P_1 - P_2)^2\right] \\ &= \int \frac{d^2\eta}{\pi} \int \frac{d^2\xi}{A\pi} \exp\left(\frac{iC\eta_1^2}{2A}\right) \left[\frac{1}{2} |\eta\rangle \langle\xi| e^{\frac{\xi\eta^* - \xi^*\eta}{2}}\right] \exp\left(-\frac{iB\xi_2^2}{2A} + \frac{\xi\eta^* - \xi^*\eta}{2} \left(\frac{1}{A} - 1\right)\right), \end{aligned} \quad (30)$$

in which

$$|\eta\rangle \langle\xi| e^{\frac{\xi\eta^* - \xi^*\eta}{2}} = 2\pi^2 \delta\left(\eta_2 - \frac{P_1 + P_2}{\sqrt{2}}\right) \delta\left(\eta_1 - \frac{Q_1 - Q_2}{\sqrt{2}}\right) \delta\left(\xi_2 - \frac{P_1 - P_2}{\sqrt{2}}\right) \delta\left(\xi_1 - \frac{Q_1 + Q_2}{\sqrt{2}}\right),$$

it then follows from

$$\delta\left(\eta_1 - \frac{Q_1 - Q_2}{\sqrt{2}}\right) \delta\left(\xi_2 - \frac{P_1 - P_2}{\sqrt{2}}\right) = \frac{1}{2\pi} \mathfrak{P} \left[e^{i\left(\xi_2 - \frac{P_1 - P_2}{\sqrt{2}}\right)\left(\eta_1 - \frac{Q_1 - Q_2}{\sqrt{2}}\right)} \right] \quad (31)$$

and Eq. (20) we have

$$\begin{aligned} (26) &= \frac{1}{2\pi A} \int d^2\eta \int d^2\xi e^{\frac{iC\eta_1^2}{2A}} e^{\frac{-iB\xi_2^2}{2A}} \delta\left(\eta_2 - \frac{P_1 + P_2}{\sqrt{2}}\right) \mathfrak{P} \left[e^{i\left(\xi_2 - \frac{P_1 - P_2}{\sqrt{2}}\right)\left(\eta_1 - \frac{Q_1 - Q_2}{\sqrt{2}}\right)} \right] \\ &\quad \times \delta\left(\xi_1 - \frac{Q_1 + Q_2}{\sqrt{2}}\right) e^{i(\eta_1 \xi_2 - \eta_2 \xi_1)\left(\frac{1}{A} - 1\right)} \\ &= \sqrt{\frac{1}{AD}} e^{-\frac{iB}{4D}(P_1 - P_2)^2} \\ &\quad \times \mathfrak{P} \exp \left\{ -i(P_1 \ P_2) \begin{pmatrix} \frac{1}{2A} + \frac{1}{2D} - 1 & \frac{1}{2A} - \frac{1}{2D} \\ \frac{1}{2A} - \frac{1}{2D} & \frac{1}{2A} + \frac{1}{2D} - 1 \end{pmatrix} \begin{pmatrix} Q_1 \\ Q_2 \end{pmatrix} \right\} e^{\frac{iC}{4D}(Q_1 - Q_2)^2} \\ &= (25) \end{aligned} \quad (32)$$

In summary, By virtue of the integration method within \mathfrak{P} -ordered product of operators and the property of entangled state representation, we reveal new physical interpretation of the generalized two-mode squeezing operator (GTSO), and find it be decomposed as the product of free-space propagation operator, single-mode and two-mode squeezing operators, as well as thin lens transformation operator. This decomposition provides experimentalists with a new scheme for generating various two-mode squeezed states, which may have potential application in quantum information and high-precision quantum metrology.

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